RB-59-7

RESEARCH BULLETIN

ON THE DIMENSIONALITY OF THE WAIS BATTERY FOR TWO GROUPS OF NORMAL MALES

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This research was supported by the Society for the Investigation of Human Ecology.

Educational Testing Service
Princeton, New Jersey

April 1959

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Abstract

Factor analysis is applied to two 19 by 19 intercorrelation matrices of Wechsler split-half subtest scores, in order to estimate the dimensionality of the basic battery. All of the WAIS tests except Vocabulary were given. The subjects were 228 male college or college-preparatory students.

Evidence is adduced supporting the statistical significance of 10 orthogonal dimensions within the 10-test battery studied, but the factors are not perfectly congruent with the subtest structure of the battery. Comprehension is found to involve two distinct factors, while no distinctive and significant factor is found for Object Assembly; the reliability of the latter can be accounted for by the Block Design and Picture Completion factors. An eleventh factor which can be interpreted as a weak doublet for Object Assembly is of questionable significance.

The results are consistent with the efforts of some clinical psychologists to interpret the Wechsler "psychogram" as a personality measure, provided attention is given to the individual items of the Comprehension and Picture Completion tests. The results are also consistent with prior factor studies of the Wechsler which have found only three to five factors; the large superficial difference in the results may be attributed to a limiting feature implicit in the methodology of most prior studies.

On the Dimensionality of the WAIS Battery for Two Groups of Normal Males[1]

Many persons (1, 2, 3, 4, 7, 10, 12, 14, 15, 16, 19) have applied factor analysis to the Wechsler,[2] and the consensus of most of their results has led to the prevailing psychometric view that this battery measures just three common factors. However, discordant results have recently been reported by Cohen (6, 8), who obtained evidence in support of five common factors for a series of samples spanning a wide range of ages. Earlier, Davis (9) reported a factor analytic study of the WB-I together with other test variables, in which he found ten distinct factors each of which was correlated with at least one Wechsler subtest.

In evaluating this situation it must be borne in mind that common factor analysis, using communality estimates in the diagonal of an 11 by 11 subtest intercorrelation matrix and assuming the validity of conventional factor analytic reasoning,[3] cannot require more than 7 factors under any conditions,

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[1] This research was supported by the Society for the Investigation of Human Ecology.

[2] By referring to "the Wechsler" generically, the reader will understand that we mean to include the original Wechsler-Bellevue Intelligence Scale - Form I (WB-I), its alternate form the Wechsler-Bellevue Intelligence Scale - Form II (WB-II), the synthesis of these forms into the Wechsler Adult Intelligence Scale (WAIS), the adaptation of the battery as the Wechsler Intelligence Scale for Children (WISC), as well as translations and other adaptations of the same set of 11 subtests for use in other cultures and languages. We shall often refer to specific subtests by means of the commonly used abbreviations, as follows: A = Arithmetic, BD = Block Design, C = Comprehension, D = Digit Span, DS = Digit Symbol, I = Information, MZ = Mazes (in WISC only), OA = Object Assembly, PA = Picture Arrangement, PC = Picture Completion, S = Similarities, and V = Vocabulary. It may be noted that tests commonly classified as "verbal" have single-letter abbreviations, while those commonly classified as "performance" have two-letter abbreviations. Descriptions of the various versions of these tests may be found in the appropriate manuals for test administration (22, 23, 24, 25).

[3] It will be evident from what follows in this paper that we may accept the validity of Guttman's (13) contention that the null hypothesis for factor analysis may well be that there are many factors, rather than that there are few. The fact remains that one's chances of separating any number of factors improve with the use of more variables, and this course is simply made more imperative by Guttman's argument.

and can provide a very good fit to the observed correlations with even fewer factors. It is therefore reasonable to suspect that the typical three-factor result may depend more on this limiting feature of the methodology rather than on any reality of the data. Cohen's results provide support for such a hypothesis, even his five-factor results having been obtained from matrices that could require eight at most. (His WISC matrices were 12 by 12.) Davis' results also support such a hypothesis, but they are not fully convincing, both because other non-Wechsler tests were a part of the analysis, and on other more technical grounds.[4] Still further support for the hypothesis is implicit in the conviction held by many clinicians that pattern analysis of the Wechsler "psychogram" works (e.g., 11, 18), even though psychometric analyses of the battery have provided virtually no support for this conviction. In a direct test of clinical pattern analysis (5), Cohen's results are again relatively optimistic, but hardly encouraging.

Being guided by the hypothesis that the Wechsler may provide meaningful measurement along substantially more than three dimensions, we were led to collect and analyze new empirical data in a manner that is capable of establishing at least as many dimensions as there are tests, but that does not force this result. Only an analysis meeting these requirements of experimental design can provide cogent evidence of the true dimensionality of such a battery as the Wechsler. It is the purpose of this report to describe such an analysis and its results.

The basis of our analytic procedure is to score as many as possible of the subtests as split-halves. This results in almost doubling the number of variables without recourse to tests extraneous to the Wechsler, and sacrifices only a portion of the reliability level of the variables; the latter effect is compensated by using a sufficient number of cases. If the test battery

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[4] Only 202 of Davis' 356 subjects took the WB-I, and each correlation in the matrix that he factored was based on all the available pairs of cases. Apparently the 202 are not a representative sub-sample, for this has led to production of a markedly non-Gramian matrix, serious distortion of communality estimates, and possible over-estimation of the number of meaningful factors.

has been successfully constructed, this procedure brings all the heretofore specific factor variance into the common factor space, and leads to the expectation that we should find a different doublet factor corresponding to each of the original subtests. That this result is not forced, however, may most easily be seen in the fact that it is not even attained.

Procedure

Data were available to us for two distinct samples who had been tested using the same slightly modified form of the WAIS. Sample A was composed of 96 male high school seniors, who had been systematically chosen from a much larger number so as to represent a wide variety of personality types (as determined by a paper-pencil inventory) and levels of academic over- and under-achievement. Sample B was composed of 132 cases, and included every male student registered as attending a particular coeducational, denominationally affiliated, four-year liberal arts college. All cases in Sample A were tested by the same experienced examiner, while all cases in Sample B were tested by another, initially inexperienced examiner.

The test battery employed with both samples was based most directly on the WAIS. However, the Vocabulary test was omitted altogether (to save time), and the remaining ten tests were administered in the sequence of and using the directions called for by WB-I. Also, additional relatively more difficult items were used with the Information, Digit Span and Arithmetic tests in order to provide adequate ceiling for each test separately for each of our subjects, and these additional items were counted in scoring when the subject got them right.

As the major innovation in our procedure, each test (except Digit Symbol) was scored as two experimentally independent parts. Wherever possible, the items of the full form were arbitrarily divided into odd- and even-numbered sub-subtests; in the case of Digit Span the separate scores were for "forwards" and "backwards." This scoring procedure yielded 19 variables, with a numerical raw score for every variable for every case in each sample. It may be noted that factor analysis of a 19 by 19 correlation matrix can theoretically yield as many as 14 factors, and that this potentiality has been provided without bringing in any new tests.

Samples A and B were first analyzed separately. Table 1 gives both of these intercorrelation matrices, along with descriptive statistics for both samples on the 19 variables. An electronic computer was used for the factoring, and each analysis was reiterated several times in order to determine the approximate number of factors and obtain stable communality estimates based on these factors. Using "rule of thumb" procedures to fix the number of factors, Sample A required a minimum of 9 factors (after six iterations) and Sample B a minimum of 10 factors (after eight iterations).

Tucker's procedure (20) for maximizing the congruence of two factor matrices was now employed to provide a statistical measure of the number of factors common to both samples. This procedure yields a series of numbers that may be regarded as correlations between corresponding factors in the two samples. When one of these correlations is sufficiently large it serves to establish the significance of an additional factor match. By trying to match too many factors, one obtains some match-correlations that are at a chance level and can recognize those which are better than chance. Accordingly, matches were sought using the first 13 factors existing after communality stabilization in each sample, and the values shown in the first column of Table 2 were obtained.

The first seven values in this column differ from unity only as a result of rounding errors in the computation; the remaining values have been arranged in order of decreasing magnitude. The second column of Table 2 shows the cumulative sums of the values in the first column, starting at the bottom of the series, while the third column shows the expected value of this sum. The latter values are expressed in fractional form, since they are exactly determined by theory. The final column gives the ratio of the observed sum to the expected sum. These ratios all have an expected value of one, and have been found in empirical studies based upon synthetic random data to behave very much like F-ratios (21), although the proper number of numerator degrees of freedom has not been worked out. (The denominator degrees of freedom are infinite, since the expected sum was given by theory.)

If we take the numerator degrees of freedom to be approximately the same as the numerator of the expected value fraction, the ratios for 8

Table 1.

INITIAL INTERCORRELATION MATRICES*

	Sample A μ	Sample A σ	I_O	I_E	C_O	C_E	D_F	D_B	A_O	A_E	S_O	S_E	PA_O	PA_E	PC_O	PC_E	BD_O	BD_E	OA_O	OA_E	DS	Sample B μ	Sample B σ
I_O	11.80	1.89	--	6133	2291	3021	1110	0158	2755	3712	2497	2545	1291	1491	1211	0508	1733	1477	1636	0618	-0568	11.46	2.30
I_E	11.51	1.94	5580	--	2501	2177	3362	1521	3002	3832	2606	2405	2646	1683	1493	1385	1812	1518	1502	1035	0932	10.44	2.12
C_O	11.69	1.64	2327	2996	--	3431	0134	-0902	0575	3000	2865	1831	0628	0573	1833	0194	-0044	-0028	0381	-0036	-0205	10.83	2.00
C_E	10.39	1.54	2117	1950	-0101	--	1241	-1518	1779	2147	2075	3305	2149	2145	1748	1412	1664	1335	0883	0532	0680	10.10	1.93
D_F	7.10	1.18	1543	2366	2930	2130	--	4318	3494	3380	0653	0479	1425	0303	0193	1070	1765	2865	1193	1276	0781	6.94	1.33
D_B	5.22	1.33	0792	1099	1366	0806	5900	--	2876	1553	-0516	-1070	0206	-0232	1174	2207	1029	1564	0661	1563	1228	6.03	1.49
A_O	7.11	1.63	2605	3562	2828	3256	4168	3669	--	5034	0184	1053	2342	1833	-0586	1040	1615	1833	1982	1666	1579	6.77	1.77
A_E	7.49	2.00	4360	3889	1901	3438	3279	2999	5636	--	2153	2245	2141	1721	0878	0759	3749	2334	1790	2657	0889	6.40	1.88
S_O	8.69	2.23	3480	2554	2586	0289	0718	0790	2471	2465	--	4331	1129	1442	1777	1469	1558	1351	1811	0108	1316	8.36	1.92
S_E	9.07	1.95	2414	2434	2524	0703	2374	0942	1710	0844	4984	--	1354	2141	1793	0954	2109	1815	2414	0259	1049	8.55	2.20
PA_O	14.29	2.87	2178	1601	-0516	2026	1081	0323	1506	1873	0483	-0112	--	3298	2608	2197	2126	3191	2189	1706	2401	12.48	3.73
PA_E	12.22	3.32	1511	1683	2829	2110	1779	0692	1951	2785	3152	2340	3131	--	2926	2107	2482	2106	1177	0303	0915	11.43	3.67
PC_O	8.24	1.47	0770	-0246	2565	0924	0518	1115	2100	2400	0482	0739	0254	3069	--	4394	2951	2997	1770	2118	1507	7.92	1.58
PC_E	7.76	1.72	1739	-0724	0436	2616	1560	-0134	0838	1820	-1006	-0382	0393	1382	3390	--	2607	3674	2340	3070	0698	7.85	1.70
BD_O	20.09	3.67	2860	0911	0031	2250	1182	1320	0816	3325	0823	-0082	4159	3216	2253	-0326	--	5250	3602	5396	0555	21.20	3.13
BD_E	18.48	4.55	2726	0124	0369	3001	1794	1958	1708	4025	2496	1018	3681	3287	2414	2311	5676	--	4524	4787	1420	19.83	3.87
OA_O	14.54	3.10	2248	0545	0108	2877	0102	0193	0888	1120	2279	-0152	3525	2598	2576	3170	3637	4876	--	4870	2828	14.83	2.91
OA_E	17.78	5.64	1933	0606	0807	0480	1996	0826	1216	2395	2990	1209	1635	2760	3443	3460	3092	5325	3619	--	1547	18.58	5.50
DS	57.72	9.22	-0534	1890	0882	1533	0430	1314	2078	1188	1111	0771	0389	0030	1918	0390	1272	1617	2134	1876	--	61.54	11.16

*Sample A values are below the diagonal; N = 96.

Sample B values are above the diagonal; N = 132.

Table 2.

FACTOR MATCHING COEFFICIENTS

#	ϕ^2	$\Sigma\phi^2$	$E(\Sigma\phi^2)$	Ratio
1	1.0002			
2	1.0001			
3	1.0001			
4	1.0001			
5	.9999			
6	.9998			
7	.9997			
8	.9339	3.2151	36/19	1.697
9	.7917	2.2812	25/17	1.551
10	.6267	1.4895	16/15	1.396
11	.5126	.8628	9/13	1.246
12	.3330	.3502	4/11	.963
13	.0172	.0172	1/9	.155

and for 9 factors exceed the conventional 5% percentage point appearing in F-tables. This appears to establish the statistical significance of at least 9 factors that must be present in both Samples A and B. Since the ratio falls below one after the 11th factor, we may accept 11 as a maximum estimate of the true number of factors common to both samples.

One way to see what the matching factors are is to proceed along the main line of Tucker's matching procedure (20), locking the two factor matrices together in their maximally congruent relation and then rotating the resulting 38-variable factor matrix towards simple structure. While this would have required less additional computation than the plan actually followed, the alternative plan that was followed offered the advantages of providing the clearest single picture of each of the factors and of simplifying the whole presentation of the final results.

The procedure that was followed was to combine Samples A and B into a single group of 228 cases, compute a single correlation matrix for the 19 variables (Table 3), and factor this matrix. In this factoring, it was assumed that there should be 11 factors, and a total of 16 iterations[5] were carried out with successively improved communality estimates, starting with initial communalities of zero for all variables. The degree of communality stabilization attained may be seen in Table 4, which shows each of the last five sets of h^2 estimates, including those generated by the final iteration. The unrotated factors obtained at this point are shown in Table 5, in order of decreasing contribution to variance.

Table 5 was rotated by machine according to Kaiser's normal varimax criterion for simple structure (17), resulting in the final values shown in Table 6. The factors in Table 6 have again been arranged in order of decreasing contribution to variance, and reflected so as to exhibit a maximally positive manifold. Parentheses have been used in Table 6 to identify the two highest loadings for each factor, and any other loadings of comparable magnitude.

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[5] The last 10 of these iterations and the subsequent rotation were carried out in a single 20-minute period on the University of California Computing Center's IBM 701, with the cooperation of Mr. Jack O. Neuhaus.

Table 3.

INTERCORRELATION MATRIX FOR COMBINED GROUPS

	N = 228 μ	σ	I_O	I_E	C_O	C_E	D_F	D_B	A_O	A_E	S_O	S_E	PA_O	PA_E	PC_O	PC_E	BD_O	BD_E	OA_O	OA_E	DS
I_O	11.61	2.15	--	5919	2412	2774	1303	0151	2754	3986	2901	2567	1712	1568	1124	0944	2005	1808	1814	1049	-0686
I_E	10.89	2.11	5919	--	3078	2220	3051	0592	3348	4257	2670	2625	2774	1893	1055	0452	0944	0465	0948	0652	0763
C_O	11.19	1.90	2412	3078	--	2402	1213	-0707	1553	2993	2807	2272	0817	1562	2257	0220	-0367	-0221	0161	0117	-0249
C_E	10.22	1.78	2774	2220	2402	--	1585	-0906	2351	2726	1384	2487	2237	2197	1524	1821	1713	1821	1587	0450	0797
D_F	7.01	1.27	1303	3051	1213	1585	--	4531	3782	3370	0721	1236	1425	0911	0379	1242	1375	2244	0713	1506	0532
D_B	5.69	1.48	0151	0592	-0707	-0906	4531	--	2768	1257	-0165	-0648	-0461	-0190	0822	1293	1520	2061	0576	1407	1674
A_O	6.91	1.72	2754	3348	1553	2351	3782	2768	--	5322	1246	1403	2227	1966	0546	0926	1072	1576	1466	1398	1538
A_E	6.86	2.01	3986	4257	2993	2726	3370	1257	5322	--	2416	1921	2560	2354	1727	1112	2943	2561	1306	2251	0468
S_O	8.50	2.06	2901	2670	2807	1384	0721	-0165	1246	2416	--	4616	1023	2232	1269	0305	1049	1770	1986	1381	1053
S_E	8.77	2.11	2567	2625	2272	2487	1236	-0648	1403	1921	4616	--	1140	2318	1505	0391	0937	1237	1296	0543	0713
PA_O	13.25	3.51	1712	2774	0817	2237	1425	-0461	2227	2560	1023	1140	--	3385	1987	1414	2336	2765	2446	1425	1224
PA_E	11.76	3.55	1568	1893	1562	2197	0911	-0190	1966	2354	2232	2318	3385	--	3059	1774	2552	2379	1693	1201	0394
PC_O	8.05	1.54	1124	1055	2257	1524	0379	0822	0546	1727	1269	1505	1987	3059	--	3932	2412	2509	2042	2564	1432
PC_E	7.81	1.71	0944	0452	0220	1821	1242	1293	0926	1112	0305	0391	1414	1774	3932	--	1266	3033	2713	3247	0615
BD_O	20.73	3.41	2005	0944	-0367	1713	1375	1520	1072	2943	1049	0937	2336	2552	2412	1266	--	5578	3639	4358	1100
BD_E	19.26	4.22	1808	0465	-0221	1821	2244	2061	1576	2561	1770	1237	2765	2379	2509	3033	5578	--	4696	5063	1719
OA_O	14.71	2.99	1814	0948	0161	1587	0713	0576	1466	1306	1986	1296	2446	1693	2042	2713	3639	4696	--	4333	2578
OA_E	18.25	5.57	1049	0652	0117	0450	1506	1407	1398	2251	1381	0543	1425	1201	2564	3247	4358	5063	4333	--	1759
DS	59.93	10.55	-0686	0763	-0249	0797	0532	1674	1538	0468	1053	0713	1224	0394	1432	0615	1100	1719	2578	1759	--

Table 4.

LAST FIVE h^2 ESTIMATES

	12th	13th	14th	15th	16th
I_O	7229	7287	7339	7383	7421
I_E	7305	7327	7342	7351	7355
C_O	3704	3683	3669	3659	3654
C_E	4649	4744	4830	4908	4981
D_F	6124	6149	6166	6178	6187
D_B	6651	6730	6800	6862	6919
A_O	8131	8322	8494	8652	8798
A_E	6358	6256	6161	6076	6002
S_O	5903	5982	6057	6126	6191
S_E	4077	4015	3961	3915	3876
PA_O	4645	4660	4671	4678	4683
PA_E	5931	5979	6003	6011	6006
PC_O	5408	5361	5315	5272	5231
PC_E	5433	5483	5529	5572	5612
BD_O	6184	6171	6156	6139	6123
BD_E	6259	6251	6246	6245	6245
OA_O	4658	4637	4621	4610	4601
OA_E	5736	5757	5768	5772	5771
DS	3451	3483	3517	3552	3587
Σ	10.7836	10.8277	10.8645	10.8961	10.9243

Table 5.

FINAL UNROTATED FACTOR MATRIX

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	h_{11}^2
I_O	5485	-4003	-1073	1830	-0671	2370	3635	-0766	1044	-1362	-0890	7421
I_E	5476	-5115	0076	-0216	-0359	1733	1330	-3165	-0491	0718	1294	7355
C_O	3072	-3557	-1698	-1396	0128	-1657	0409	1393	-1338	-0010	1722	3654
C_E	4294	-1626	-1545	-0174	-2277	-1172	-1763	1332	3190	-1720	1319	4981
D_F	4294	-0991	4681	-2948	1023	1210	-0865	0795	1698	1823	1321	6187
D_B	2366	1799	5963	-3401	2688	1008	0878	-0689	0114	-1741	-0840	6919
A_O	5068	-2948	5787	2195	-1580	-2898	-1213	0436	-0671	-0240	-1494	8798
A_E	6315	-2565	1486	0232	-1117	0639	-0682	2077	-2178	0328	-0113	6002
S_O	4277	-2197	-2784	0842	4991	-1759	0072	0535	-0085	0749	-1214	6191
S_E	3792	-2359	-2497	-0017	2805	-1435	-0741	0664	1220	-0092	-0399	3876
PA_O	4494	0270	-1227	-0161	-2417	0794	-3186	-2132	-0390	1327	-1394	4683
PA_E	4141	0882	-4295	-3729	0227	2282	-1892	-0164	-0421	0078	-0860	6006
PC_O	4200	2169	-2164	-2995	-1246	-2416	1362	-0026	-2193	-1500	0032	5231
PC_E	3679	3230	-0411	-2431	-2424	-2631	2893	-0139	1612	1121	-1016	5612
BD_O	5247	3888	-0266	1869	0073	2953	-0655	1502	-0845	-1647	0428	6123
BD_E	5898	4721	0174	1401	0534	1077	-0240	0923	0937	0185	-0333	6245
OA_O	4794	3520	-0845	2291	0464	-0905	0077	-1585	0963	0350	0271	4601
OA_E	4845	4587	0431	1720	0494	-0381	1709	0435	-1102	1792	1457	5771
DS	2227	1864	0896	0477	1472	-2327	-1936	-3295	-0400	-1520	1324	3587
Σa^2	3.9248	1.7767	1.4019	.7391	.6680	.6404	.5339	.4191	.3451	.2608	.2146	10.9243

Table 6.

ORTHOGONAL ROTATION ACCORDING TO NORMAL VARIMAX CRITERION

	I (BD)	II (I)	III (S)	IV (D)	V (A)	VI (PC)	VII (PA)	VIII (C)	IX (C')	X (DS)	XI (?)	h_{11}^2
I_O	1618	(7635)	2275	-0056	1292	0621	0219	1518	0058	-1465	-1256	7421
I_E	-0262	(7303)	1779	1505	1460	-0197	2161	0495	2079	1037	(1498)	7355
C_O	-0906	1884	2903	-0085	0939	0864	0416	1490	(4419)	-0269	0340	3654
C_E	0911	1416	1476	-0125	1210	1106	1475	(6222)	1045	0333	0011	4981
D_F	1062	1105	0575	(6799)	1914	0089	1028	1415	0964	-0321	(2281)	6187
D_B	1119	0123	-0571	(7474)	0909	0995	-0765	-1475	-0683	1558	-2068	6919
A_O	0728	1882	0662	2423	(8579)	0308	-0212	1276	0406	1433	0157	8798
A_E	2453	2813	1602	1849	(4510)	-0062	2251	1088	(3462)	-1174	-0372	6002
S_O	1264	1390	(7536)	-0087	0461	0052	0439	-0377	0805	0625	-0003	6191
S_E	0436	1285	(5622)	0117	0281	0205	0922	(1859)	0801	0476	0000	3876
PA_O	1821	1394	0245	-0382	1816	0812	(5838)	1087	-0165	1252	0739	4683
PA_E	1660	0659	2345	0829	-2819	1468	(5875)	1069	1714	-0519	-1314	6006
PC_O	1914	0207	0883	-0031	-0257	(5140)	1783	0302	(3376)	1510	-2097	5231
PC_E	2191	0210	-0103	0873	0264	(6960)	0651	0980	-0288	-0059	0723	5612
BD_O	(7123)	0742	-0074	0859	0153	-0461	1739	0999	0698	0105	-2108	6123
BD_E	(7158)	0051	1059	1560	0575	1641	1568	1040	-0852	0540	-0262	6245
OA_O	(5294)	0894	1432	-0475	0448	1976	0918	0694	-1174	(2720)	0840	4601
OA_E	(6705)	0228	0410	0586	0622	2420	-0247	-1111	1070	1029	(1564)	5771
DS	1478	-0385	0657	0749	0595	0343	0482	0193	-0015	(5639)	-0077	3587
Σa^2	2.0673	1.3746	1.2112	1.1958	1.1753	.9399	.9368	.6128	.5796	.5466	.2844	10.9243

Discussion

We know, on the basis of the matching results for Samples A and B, that there must be a minimum of 9 significant factors in Table 6. There may be more -- partly because the significance test was one requiring positive evidence for acceptance of each factor, and partly because there may have been a factor in either sample that could not be matched in the other. However, the matching results also established 11 as a reasonable upper bound on the number of factors, and this number was used to obtain Table 6.

Examination of the results in Table 6 suggests that the 11 factors may be divided into three groups. The first group would include Factors I through VII; there can be no question about either the statistical significance or the psychological interpretation of any of these seven factors. Each of these factors can be clearly identified by its two highest loadings, which are derived from the two parts of what is normally the same test. These seven factors appear to be relatively independent of item content, and to depend primarily on item form, or item type. Except for Factor I, which brings BD and OA together, these results have little to contribute to our understanding of the meaning of these item forms. Factor I has been identified as BD because the BD loadings are both larger than both OA loadings, and BD thus appears to provide the better definition of the factor.

The second group of factors would include Factors VIII, IX, and X, while the third group would include only Factor XI. In the varimax rotation, Factors VIII, IX, and X are all of the same order of magnitude of importance, making it virtually impossible to regard only some of them as significant. Since we know from the matching results that at least two of these are significant, we are forced to conclude that all three of them are; this brings the total number of significant factors up to ten. On the other hand, while Factor XI may reflect a true dimension, only a relatively weak case may be made in favor of its statistical significance.

Factor VIII has a loading for the even Comprehension score that is of the same order of magnitude as the large loadings appearing on Factors I

through VII, but the odd Comprehension score has only its fourth highest loading on this factor. Thus, while this factor comes the closest to providing a doublet for C, and has been so identified, it appears to depend more on item content than on item form. Consideration of the differential item content of C_O and C_E, and of the item content of S_E, which provides the second highest loading for this factor, suggests that this factor probably measures the subjects' conventional understanding of certain basic principles affecting interpersonal relations. Thus, subjects who score high on this factor must tend to respond correctly to items which require them to recognize the influence of bad company, to understand the function of government taxation and regulation of labor and marriage, and to relate "praise" and "punishment" in terms of their influence on future behavior. It may be noted that all the C items judged to be relevant for this factor appear in both WB-I and WAIS, but not in WB-II.

Factor IX is marked by three loadings, all of a somewhat lower order of magnitude and all from parts of different tests. Again it seems clear that we are dealing with a content factor; we have identified it as C' solely because the even Comprehension score seems to provide the cleanest available measure of the dimension. Examination of the differential item content of PC_O and PC_E suggests the relevance of distinguishing between responses that depend on fairly specific prior experience (i.e., learning) and other responses that depend solely on noticing the relevant stimulus information in the PC item (i.e., recognition of incongruity). The latter function may be attributed to Factor VI, and the former to the present Factor IX. "Breadth of Experience" provides an interpretation of this factor that appears consistent with the loadings (and atypical item content) of C_O and A_E, but this interpretation might also be construed to call for loadings from I_O and/or I_E. Since Information does not load this factor, it seems best to regard Factor IX as "Breadth of Practical Experience," reserving the concept of "Breadth of Intellectual Experience" for possible application to Factor II.

Factor X has a good loading for DS, and since it was not possible to split this test into operationally independent parts, it is unreasonable to expect any clear doublet to emerge for Digit Symbol. It may well represent

a distinct dimension solely as a function of its item-type. Thus, while we have identified this factor as DS, it seems likely that the inclusion of a parallel form of the Digit Symbol subtest would result in some changes; the loading for DS would probably be higher, and that for OA_O could be lower without loss of common-factor status for the dimension.

Factor XI poses the only difficult problem in interpretation. There seem to be at least four distinguishable possibilities, as follows:

(1) The factor may be pure chance. There is no statistical evidence that will clearly establish the significance of this factor. It has no large loadings, and only accounts for a small amount of variance. It is virtually identical with Factor X of the unrotated matrix, not having participated very much in the varimax rotations.

(2) The loadings may represent the near hyperplane of a Vocabulary factor, which would probably have been found if the Vocabulary test had been given to our subjects and included in the analysis in the same fashion as the remainder of the Wechsler battery. Any of the first seven factors would look much like Factor XI if its marker test had been omitted from the battery.

(3) The difference between the loadings for D_F and D_B may be meaningful; a small rotation against Factor IV would give a loading of .4 for D_F while putting D_B in the hyperplane. A similar rotation against Factor I would bring the Block Design scores into better alignment with the hyperplane, and create loadings for OA_O and OA_E of about .2 and .3, respectively. These rotations would also increase the variance accounted for by the factor, but not by enough to remove it from last place in the series.

(4) Capitalizing on the same differences in the profile of loadings, the factor might be reflected and rotated against Factors I and IV to produce the largest positive loadings on D_B and BD_O.

While any attempt to apply the results of this study practically must obviously proceed on the basis of Hypothesis 1 above, there are several attractive features of Hypothesis 3. This hypothesis would lead to identification of the factor as OA, and such a factor is otherwise missing from our results. OA scores that are widely divergent from a Wechsler profile are

sometimes interpreted (11) as betraying a divergent level of anxiety within the subject (low OA corresponds to high anxiety). Divergent scores on Digit Span, and particularly D_F, have sometimes been given similar interpretations (11, 18). The extremes of anxiety, however, are found primarily in "abnormal" populations. Since there are very few clinically sick individuals in our Samples A and B, this hypothesis as to the meaning of Factor XI would account for its relatively small variance. Also, as was noted under Factor X, the lack of a parallel form for DS may have helped Factor X to steal some OA variance that could belong to Factor XI.

In any event, however, further empirical data should be gathered and analyzed if the status of Factor XI is to be clarified. The battery used in the present analyses should be augmented with a parallel form for DS and two Vocabulary subscores. The sample should include a significant number of abnormal personalities, as well as representative cross-sections of normal groups.

Summary

Factor analysis is applied to two 19 by 19 intercorrelation matrices of Wechsler split-half subtest scores, in order to estimate the dimensionality of the basic battery. All of the WAIS tests except Vocabulary were given. The subjects were 228 male college or college-preparatory students.

Evidence is adduced supporting the statistical significance of 10 orthogonal dimensions within the 10-test battery studied, but the factors are not perfectly congruent with the subtest structure of the battery. Comprehension is found to involve two distinct factors, while no distinctive and significant factor is found for Object Assembly; the reliability of the latter can be accounted for by the Block Design and Picture Completion factors. An eleventh factor which can be interpreted as a weak doublet for Object Assembly is of questionable significance.

The results are consistent with the efforts of some clinical psychologists to interpret the Wechsler "psychogram" as a personality measure, provided attention is given to the individual items of the Comprehension and Picture Completion tests. The results are also consistent with prior factor studies of the Wechsler which have found only three to five factors; the large

superficial difference in the results may be attributed to a limiting feature implicit in the methodology of most prior studies.

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